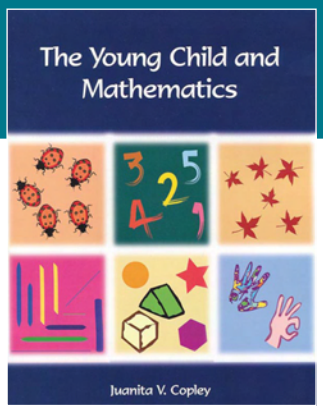
Developmentally Appropriate Practice

A reading from the CD accompanying *Developmentally Appropriate Practice in Early Childhood Programs Serving Children from Birth through Age 8, Third Edition.*

READING #34

Geometry and Spatial Sense in the Early Childhood Curriculum



Juanita V. Copley

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Chapter 6

Geometry and Spatial Sense in the Early Childhood Curriculum

Geometry is the area of mathematics that involves shape, size, position, direction, and movement and describes and classifies the physical world we live in. Children's *spatial sense* is their awareness of themselves in relation to the people and objects around them.

Historically, geometry was one of the first areas of mathematics taught to young children. In the 1850s Friedrich Froebel, "the Father of Kindergarten," designed a curriculum with suggested instructional practices based on the use of geometric forms and their manipulation in space. In this curriculum Froebel designed "gifts" for kindergartners—special materials to enable them to explore and grasp basic forms and relationships. The first six gifts included balls of different colors, cubes, spheres, cylinders, and complex sets of geometric blocks that children manipulated and observed in a series of progressive tasks (Balfanz 1999).

In later years this geometric focus was largely lost. In fact, in an international comparison the United States's worst performance was in geometry (Beaton et al. 1996). Many reasons have been suggested for this dismal showing; however, everyone acknowledges that the study of geometry and spatial sense has not been a focus in the typical elementary or secondary school mathematics curriculum in the United States. This is especially true for early childhood classrooms. Shape definitions are typically the only prominent geometric ideas introduced, while manipulation of shapes and spatial exploration are generally neglected. Young children have worked with shapes in art activities and puzzles, and have constructed with Legos and unit blocks, which offer rich opportunities to explore geometry and spatial relationships. However, many teachers do not emphasize spatial concepts nor take advantage of natural connections to mathematics or other content areas.

As discussed here, the content standard on geometry and spatial sense involves much more than naming shapes. Key aspects of geometry and spatial sense, according to the NCTM standards for the early grades (2000), are

• analyzing characteristics and properties of two- and three-dimensional geometric shapes and considering geometric relationships,

• specifying locations and describing spatial relationships using coordinate geometry and other representational systems,

• applying transformations by recognizing and applying slides, flips, and turns as well as recognizing and creating shapes containing symmetry,

• using visualizations to create mental images of geometric shapes using spatial memory; to recognize and represent shapes from different perspectives; and to recognize geometric shapes and structures in the environment and specify their location. (p. 97)

Familiarity with shape, structure, location, and transformations and development of spatial reasoning enable children to understand not only their spatial world but also other mathematics topics. As children count the sides of two-dimensional shapes or the faces of a cube, they learn about number relationships. Patterns, functions, and even rudiments of algebra may be noted when children identify patterns in space or when they see the relationships between the number of faces, edges, and vertices of three-dimensional figures. When children compare shapes, directions, and positions in space, they develop concepts and acquire vocabulary that they also put to use in measurement. Grouping items, sometimes by shape or another geometric feature, is a skill also fundamental to data collection, and children may record and report shapes in an activity or in the environment.

Spatial sense and construction come into play in art, science, social studies, movement and music, and reading. For example, spatial thinking skills emphasized in geometry are critical to the making and reading of maps—essential skills in social studies. Children notice shapes in natural objects of all kinds. They discover many things about shape and geometry in their block play. Manipulating shapes in space introduces children to vocabulary words about position as well as other words necessary for reading and language arts. Even distinguishing between letters of the alphabet involves attention to shape and position. In art, spatial relationships and geometric forms are critical elements in both two-dimensional and three-dimensional creations.

Young children enjoy manipulating shapes in space, and their spatial capabilities often exceed their numerical skills (NCTM 2000). Three-year-old Jeffrey in Chapter 2 has a strong intuitive knowledge about shapes and how they relate to his world. While most of his knowledge is perceptual in nature, he is able to relate a two-dimensional circle to his uncle's basketball and to recognize that, unlike a basketball, the circle will not bounce.

As in the other content areas, the teacher's role is to bridge the young child's informal knowledge and formal school mathematics. This bridging often means using the child's own language and relating it to formal terms and definitions. It may also mean offering a position or shape word that describes what the child is doing or attending to. Jeffrey's teacher, for example, later referred to "the pizza shape," as Jeffrey called it, and compared it to a triangle. The teacher might decide to provide the word *sphere* for the bas-

ketball and engage Jeffrey in considering other differences between a ball and a two-dimensional circle.

Levels of Geometric Thinking

Describing children's development of geometry and spatial sense, researchers and educators often present Pierre Van Hiele and Dina van Hiele-Geldof's Levels of Geometric Thinking (van Hiele 1986). Most children up through the primary grades have not progressed beyond Level 1, and many preschoolers are operating at Level 0. Research suggests that to move through these levels, children must be exposed to many experiences and participate in numerous activities. Progress is often very slow.

Level 0: Children learn to recognize geometric figures such as squares and circles by their holistic physical appearance. For example, a given figure is a circle because it "looks like a clock." Children at this level do not think about the attributes or properties of shapes.

Level 1: Children begin to learn isolated characteristics or attributes of the forms, such as "a square has four equal sides."

Level 2: Children establish relationships between the attributes of a form. At this level, for example, children can determine that a square is a rectangle because it has all of a rectangle's properties.

Let's look first at a thinking activity that the teacher calls Look, Make, and Fix. Using an overhead projector, the first-grade teacher, Mr. Quintanilla, shows the children a model created from several tangram pieces. They study the configuration of shapes on the screen and then are given time to duplicate the figure with their own tangram pieces, look again, and fix their models to match the teacher's figure. Mr. Quintanilla encourages the children to help one another. As the configurations increase in difficulty, he gives the children as many tries as they need to correct their figures.

Playing Look, Make, and Fix in small groups, children who have difficulty with spatial skills can have more practice and learn new strategies. The game can also be played with attribute or pattern blocks. Children enjoy the activity so much that they like to play it on their own when they have the chance.

Look, Make, and Fix

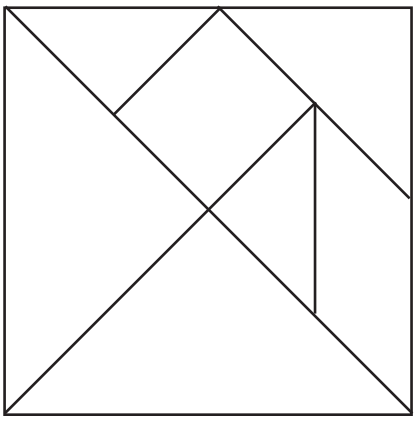
Children are getting settled in the classroom as the school day begins. The message on the chalkboard directs them to select a set of tangrams. Seeing the projector, the children anticipate one of their favorite thinking activities and excitedly get their tangram bags together—seven tangram pieces that combine to make a large square. (See the diagram on p. 108.)

Mr. Quintanilla: Today we are going to get your thinking started by playing Look, Make, and Fix. First, let's check our tangram bags to see if we have all the shapes. Who remembers what we should have in our bags?

Tanya: Two big triangles and two little triangles.

Stanley: And one medium triangle. Five triangles!

Mr. Quintanilla: Right. That covers the triangles. What about the other shapes? There are seven shapes altogether, so how many more do we need to find?



Seven tangram pieces combine to form a large square.

Jane: There's a square and that funny shaped one, a squashed rectangle. I forget.

There is some discussion as the teacher pauses, listens, and observes children finding the parallelogram. Although the children have heard the words *parallelogram, rectangle,* and *quadrilateral,* they are not sure what they mean or how they relate to each other or to the particular "squashed rectangle" in the tangram set.

Mr. Quintanilla: Let's everyone look at Jane's squashed rectangle. Hold it up, and let's trace around it with our fingers. *(modeling with his parallelogram shape as he speaks)* Side . . . stop, tip. *(touches the vertex, the point where two sides meet)* Side . . . stop, tip. Side . . . stop, tip. Side . . . stop, tip. How many sides?

Children: Four.

Mr. Quintanilla: Then, although it can be called different things, like *squashed rectangle,* let's call it a *quadrilateral.* A quadrilateral is any shape that has four sides. In fact, let's use one of Jane's words and call it a *squashed quadrilateral.* Can you say it fast? Squashed quadrilateral, squashed quadrilateral, squashed quadrilateral. *(lots of laughter)*

Mr. Quintanilla decides not to further define a quadrilateral or identify the square as a special quadrilateral. Geometry and spatial sense are skills that develop over time, and those ideas will be introduced in future lessons. For the moment, the children have the words to help them identify the shape, and that is enough.

Mr. Quintanilla: All right. Check your shapes and see if you have every shape. *(places shapes on the overhead projector for children to see as he says the words)* Five triangles, two small, two large, and one medium; and one square and one squashed quadrilateral. A total of 2 . . . 4, 5, 6, 7 pieces. Is everyone ready?

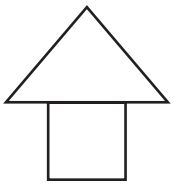
One child locates a missing triangle on the floor and another gets a square from the box of extra pieces. Everyone is now ready.

Mr. Quintanilla: Place your pieces in a pile at the top of your table. Hands in your laps! Don't move them until I tell you to. Here we go.

Using the medium triangle and the square, Mr. Quintanilla makes a shape configuration on the projector.

Mr. Quintanilla: *Look!*

Children stare at the picture for a few seconds without touching their pieces. Mr. Quintanilla then covers the picture with a piece of paper.



Mr. Quintanilla: *Make!*

Children select the pieces used and make the picture at their tables. Some look at others and copy or compare pictures. Most children make it correctly. A few use a different size triangle at the top of the picture. Mr. Quintanilla waits about 30 seconds while children continue checking, and then he removes the piece of paper covering the model.

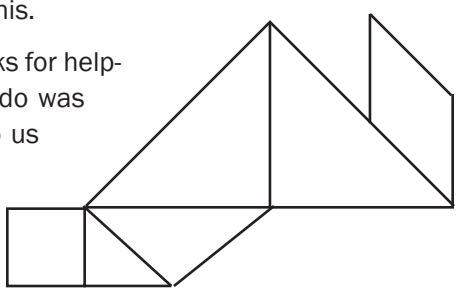
Mr. Quintanilla: *Fix!*

Children check their creations against the shape projected. Most of the children who have used the wrong size triangle see their error and change pieces. Two children do not. Mr. Quintanilla reminds children to help each other, so one child helps another with the configuration, and Mr. Quintanilla quietly changes the incorrect piece of the other child.

Kimberly: *(raising her hand)* Mr. Quintanilla, Sara is looking at my picture and copying!

Mr. Quintanilla: That's fine! Remember, this is not a test; it is a game where we all help each other. Sara, I'm glad Kimberly has a good picture for you to look at. Thanks, Kimberly! Let's try another one. This one is harder!

Mr. Quintanilla repeats the process with three more examples. They get progressively harder, each adding more shapes in different configurations. The last task takes many "fix" steps for everyone to complete. It looks like this.



Mr. Quintanilla: Whew! That last one was a lot of work. Thanks for helping each other fix that picture. I noticed that Fernando was excellent at solving this one. Fernando, can you help us understand how you did it?

Fernando, whose home language is not English, tends to have difficulty verbalizing his thoughts. After a long pause, he shrugs and responds.

Fernando: I just thunk and thunk!

Mr. Quintanilla: I could see that you did! Did you think of a picture that the shapes made? Did it look like anything to you? A *gato,* maybe?

Fernando: *(laughing)* Not a cat—a turtle with no legs!

Mr. Quintanilla: Oh, what a good idea. What about you, Gina? You helped lots of people on that one. How did you do it?

Gina: I just worked on one part at a time. First I made the mountain *(pointing to the two large triangles)* and then I added the other stuff. The hardest part was that weird piece.

Mr. Quintanilla: The squashed quadrilateral?

Gina: Yeah. I looked at that a lot before I got it.

The discussion continues a few more minutes, then the tangram pieces are put away. Mr. Quintanilla will discuss more varied spatial tasks, strategies, and geometric terms at another time. Spatial sense takes a long time to develop, and this is just a beginning exploration.

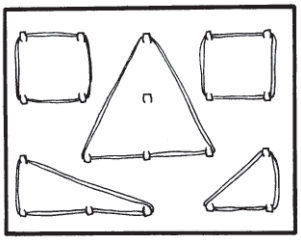
Mr. Quintanilla's class does this basic activity in varied settings—large groups, small groups, and learning centers. Families can also play Look, Make, and Fix and other games relating to space and shape. During holiday time Mr. Quintanilla sends two die-cut tangram sets home as a gift so children can teach their families the game.

An interesting sidelight is that Mr. Quintanilla himself has never found spatial sense activities easy. Many teachers with this difficulty would say to themselves, "Some people have it and some don't," and either neglect spatial activities or convey a negative, defeated attitude toward them. Instead, Mr. Quintanilla highlights the importance of effort rather than luck or intelligence in demonstrating spatial sense. He talks about how hard children are working, and he offers the "fix" component of the activity as often as needed to allow children to succeed. Believing that spatial sense activities are important for all children to experience, Mr. Quintanilla has made sure to include plenty of them and become comfortable doing them.

The main purpose in assessing children's understanding is to tailor learning experiences to their needs. Mr. Quintanilla observes children in the whole-group setting and notes those who would benefit from small-group sessions. In small groups the children can observe the configurations more closely and even handle them. In this activity the teacher notes growth in Fernando's confidence and in Gina's use of strategies and continues to delight in Jane's descriptive vocabulary.

Children at play—With geometry and spatial sense

Young children naturally love to explore geometric and spatial aspects of the world around them. There are many opportunities for the teacher to scaffold children's understanding by asking questions, suggesting other activities, showing various transformations (such as two same-size right triangles forming a rectangle), and providing additional materials.



Denise, age 4, excitedly showed her teacher some shapes she had drawn in her journal. Mrs. Stipe gave her a geoboard with bands and asked if she could copy them on her board. Denise proudly demonstrated the result.

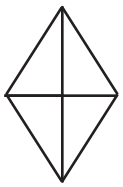
Ms. Patterson found three large plastic mirrors at a garage sale and placed them in the housekeeping center. She listened as a verbal 3-year-old gave everyone a guided tour: "Look in here. I can see two mes and two yous. These are called *magic windows.*" For weeks children experimented with their mirror images and reflections.

Creations in the block center provide many opportunities to enhance children's understanding of geometry. Questions like, "How is your tower

different from Joe's?" or "What will happen if the bottom block is removed?" or "What if I try to make a building like yours without looking? Can you tell me what I need to do to make mine just like yours?" or "It will soon be time to clean up. How will you remember what you have built?" can facilitate thinking and experimentation. As a result of the teacher's questions and discussion with 7-year-old Frank, the balance and symmetry of his block creation were enhanced and noted more consciously by Frank and the other children looking on.

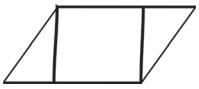
Geometry and Block Play

Using blocks presents children with many opportunities to make discoveries about two-dimensional and three-dimensional shapes. A multitude of configurations appear from children's spontaneous arrangements.



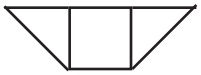
Four of the small triangles can make a square.

Four of the large triangles can make a rhombus.

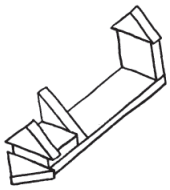


Two squares and four small triangles make a hexagon.

Two triangles joined to a square or a rectangle make a parallelogram.



Or two small triangles and a square create a trapezoid (the long side of the trapezoid may be at the top or bottom; if the long side is at the top, the children may call it a boat).



Children also make many discoveries about shape when constructing objects they use in creative play.

Adapted from E. Hirsch, ed., *The Block Book,* 3d ed. (Washington, DC: NAEYC, 1996), 57–58.

Promoting development of key skills and concepts

Although young children often demonstrate intuitive spatial abilities, many teachers virtually ignore geometry and spatial sense. The new *Principles and Standards for School Mathematics* (NCTM 2000) emphasizes geometry and spatial sense as an important content area for children in prekindergarten settings through twelfth grade. Let's look at the specific expectations for young children.

Shape

Children begin to form shape concepts in the years before school, and these concepts are fairly stable by the time children are 6 or 7. Clements (1999) suggests that an ideal period to learn about shapes is between 3 and 6 years of age. For the most part, young children do not develop their concepts of shape from looking at pictures or merely hearing verbal definitions ("a triangle has

three sides and three angles"). Rather, they need to handle, manipulate, draw, and represent shapes in a variety of ways.

The study of shapes should focus on the attributes and properties of both two- and three-dimensional shapes. Initially, children must be given many opportunities to manipulate and sort shapes according to their own criteria. Particular sorting clues can easily be highlighted as children sort blocks and put them away on shelves that are marked with outlines for each type of block or by some other system. For example, tubs can be provided for children to sort pattern or attribute blocks according to number of sides (quadrilaterals vs. triangles). Matching or classifying objects by such properties helps children to focus on the critical attributes of each shape.

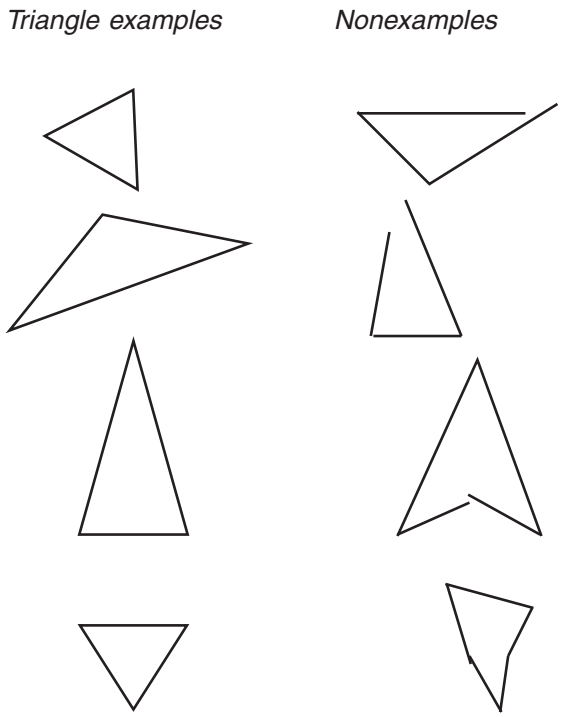
Research strongly supports the use of a wide variety of manipulatives to help children understand geometric shapes and develop spatial sense (Greabell 1978; Clements & McMillen 1996). Manipulating geometric solids helps children learn geometric concepts (Gerhardt 1973; Prigge 1978). Solid cutouts of shapes are more conducive to shape learning than are printed forms (Stevenson & McBee 1958).

Computer programs that allow children to manipulate shapes have specific advantages because, if designed correctly, they offer great flexibility (Clements & McMillen 1996). The best software allows children to instantly change the shape and the size of forms as well as save and later retrieve their work.

Beginning with shapes they can pick up and handle, children can familiarize themselves by tracing the outline of a shape with their fingers. When the child gets to the end of a side—reaches a corner or an angle—the teacher, and then the child, notes this verbally. For example, in tracing a rectangle one might say, "Side, stop, corner; side, stop, corner; side, stop, corner; side, stop, corner." By saying *corner* instead of *tip,* and by using a different or perhaps louder voice, the teacher can draw children's attention to a right angle as being different from other angles. A right triangle can be traced and described using *corner* at the right angle and *tip* for the other two angles. Children can go on to trace around cutouts or pictures of shapes.

At first, children recognize a shape by its appearance as a whole. Children may know that a triangle has three sides, and perhaps recognize the familiar equilateral triangle, yet be unable to identify triangles within a group of figures. Having a limited understanding of "triangleness," young children often do not recognize triangles that depart from the equilateral version they know and love! Four-year-old Daniel, for example, recognized the regular and irregular triangles in a group with all triangles, but he declared, "These are the really *good* triangles [pointing to the equilateral triangles resting on one of their sides] and these ones are *bad* ones [pointing to the fat, long, skinny, or irregular triangles standing on their points]!" Similarly, children see a triangle standing on its vertex as wrong and call a square sitting on its point a diamond.

Children most readily learn the critical attributes of a geometric shape when they see a variety of examples and nonexamples (Clements 1999). Examples of triangles shown in a variety of positions and sizes, as well as nonexamples (triangle-like shapes with curved sides, without sides, or with too many sides), should be shown, manipulated, and discussed. Most important, children benefit from practice in telling why a particular shape does or does not belong in a group.



At their level of development, young children do not easily categorize shapes in more than one way. Children tend to see squares and rectangles as two discrete shapes rather than seeing squares as a subset of rectangles; they expect a rectangle to have two long sides. A square is a special rectangle, of course, one that has equal sides. It is also a quadrilateral, a parallelogram, and a rhombus. Introducing all these words and concepts to young children is probably premature. However, teachers should keep the real meaning of such terms in mind and avoid giving children ideas that are actually wrong (for instance, by saying "No, that's not a rectangle—it's a square").

As Mr. Quintanilla uses the words *side* and *tip*, he models tracing the parallelograms with his finger to clarify their meanings. He discusses with the children Jane's *squashed rectangle* terminology and introduces a new word, *quadrilateral.*

Mr. Quintanilla combines tangram shapes, placing them in a variety of positions for the children to replicate. To be successful, a child must not only recognize various shapes but also note their positions in space. At this point the children are not asked to draw the shapes, but this is planned for later in the year.

Space

Thinking spatially—visualizing shapes in different positions and imagining movements—is important to young children's development as mathematical thinkers. Specifically, teachers need to help children develop "a variety of spatial understandings: direction (which way?), distance (how far?), location (where?), and representation (what objects?)" (NCTM 2000, 98). Of course, learning to think spatially is an evolutionary process. In the early childhood years, experiences with simple maps, position words, and opportunities to manipulate shapes into various positions are important to children's development of spatial sense.

These skills can be emphasized during routines or in specially planned activities. For example, children in a kindergarten class love finding their class puppet every week by reading a map the teacher gives them. The map

includes direction words and arrows, geometric shape landmarks, distance between landmarks in number of footsteps, and a pictorial representation of the puppet to be found. Later in the year, the kindergartners make the maps to help their peers find the puppet.

The creative dramatics center is a good place to emphasize spatial vocabulary. As children act out stories like "The Three Billy Goats Gruff" or "The Three Bears," position words can be used as prompts for their movements. During guided reading activities, position words can be emphasized and used in context in other meaningful situations. Also, language expressing how something is moved (right, left, up, down) and what it looks like after it is moved (standing on one tip with the big part on the top) should accompany actions that involve moving objects in space. Using children's language combined with meaningful definitions, the teacher and the children can describe how movements are made and the resulting effect.

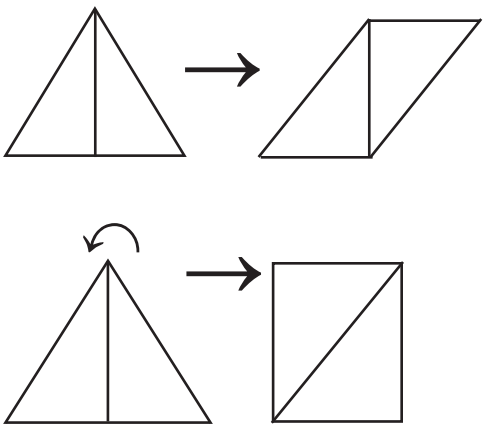
Spatial Vocabulary

Location/position words: on, off, on top of, over, under, in, out, into, out of, top, bottom, above, below, in front of, in back of, behind, beside, by, next to, between, same/different side, upside down

Movement words: up, down, forward, backward, around, through, to, from, toward, away from, sideways, across, back and forth, straight/curved path

Distance words: near, far, close to, far from, shortest/longest path

Transformation words: turn, flip, slide



Transformations. Young children are often unable to visualize what the shape will look like when it is turned, flipped, slid, or transformed in some way. When a group of 4-year-olds practice folding a square diagonally and cutting it into two triangles, they often giggle in surprise when they can move the two triangles back and make a square. Similarly, when children flip up half of a triangle and make a parallelogram, they are delighted at the transformation and may practice it over and over again to see the changes. Moving puzzle pieces and making shapes fit exactly into frames require children to transform shapes by sliding, flipping, or turning them.

Lines of symmetry. A line of symmetry divides a figure into two parts that are mirror (reversed) images of each other. To further their understanding of symmetrical relationships, children can draw half of a picture—half of a person, half of a house, half of a flower. When they place the edge of a mirror (held at a right angle to the paper) against the unfinished edge of the

image, the image will look complete. Children also enjoy holding a mirror to a family photograph, making Mom with two heads or brother Tom with four legs.

Another way for children to explore symmetrical relationships is to cut out two duplicate geometric shapes (squares are easy), then fold them each along the same line of symmetry (for squares, the line can be vertical, horizontal, or diagonal). When they hold one folded square against the other, axis to axis, children can see that the matching halves form a whole. Children also experiment with balance and symmetry while building with Legos, wooden blocks, and other materials. For young children, exploration should be the primary focus of experiences with symmetry.

Providing a mathematics-rich environment

More than any other content area, geometry requires certain specific types of manipulatives to help children learn. Attribute blocks (shapes in different colors with shape, size, and thickness attributes), pattern blocks (red trapezoids, orange squares, green triangles, yellow hexagons, and blue and white rhombi), and tangram pieces are important shape models that are helpful to children as they develop their understanding of shape and space.

Three-dimensional models are also important. A variety of wooden blocks with unusual shapes, along with the more typical shapes, help children develop their perceptions of shapes in space. Children can use clear containers (sphere, cube, cone, rectangular prisms, pyramids) filled with water to explore the water surface and provide opportunities for explorations of moving shapes.

Everyday objects also contribute to the development of children's understanding of geometry and spatial sense. The young child's world is filled with shapes in different sizes and positions. Basketballs, cereal boxes, and cans are examples of geometric models that children can compare to classroom models. Patterned material or artwork with shapes positioned in unusual ways can be displayed on classroom walls. Children can use string, pipe cleaners, and yarn to create the outlines of shapes or shape sculptures. The teacher can share books that involve children in looking at objects from different perspectives, or children can work on books of this kind on their own. Objects children bring from home to share may be excellent examples of shapes and add to a mathematically rich environment.

Young children also benefit from having many opportunities to climb in and out of big boxes, on or around equipment—going under, over, around, through, into, on top of, and out of different things to experience themselves in space. With an assortment of large hollow blocks, boards, saw horses, and the like (Cartwright 1996), children can build structures big enough to get inside, which allows them to experience their constructions from a very different spatial perspective.

Questions Relating to Geometry and Spatial Sense

Although experienced early childhood teachers ask many questions, they often overlook questions relating to spatial sense and shape. Here are some possibilities:

How is that shape like this one? How is it different?

Why isn't this shape an [oval]? What makes it a [circle]?

What if I turned this shape? What if I flipped it? What would it look like if I slid it from your paper to my paper?

Where have you seen this shape before?

Can you find something like this at home?

(When a child has made a picture out of shapes) How did you decide to use this triangle for the roof?

How did you decide what to copy/draw?

Can you tell me how to get to the [cafeteria] from here?

Can you tell me about the neighborhood you built with blocks? I'm going to draw a map of it without looking. So tell me what it looks like and what I should put where.

Do you think this shape would roll? Slide? Could we stack these?

How could you cut this paper to make another shape?

What shape could you make out of these shapes?

Could we make the cone roll straight, or would it roll crooked? What about the cylinder?

What would happen if I dropped this [cube, cylinder] and it broke in half? What would the parts look like? Could it break in half another way?

Have you found all the ways to put those shapes together? How do you know?

What would happen if I cut off an end of this? What would it look like?

Can you think of another name for this shape?

Can you make a square [a triangle, a rectangle] with pipe cleaners? How about a ball [a box, a cone]?

Some activities based on geometry and fostering spatial sense are described here. While these activities are roughly listed in order of difficulty, many can be expanded to be more challenging or can be streamlined to simplify them.

String Shapes

Three or four children hold a large string loop. They make a variety of shapes by adding or taking away a vertex or a side; changing the size of an angle, or increasing or decreasing the area of a shape.

Children enjoy making shapes as the leader of the activity names them. They find that a triangle is easy to make because it can be skinny, fat, or "just right" and still be a triangle. A square is harder because all the sides must be exactly the same. Children are often surprised that a circle is one of the hardest shapes to make with string, as it is easy to draw. As one child explained, "Circles are easy to draw because you don't have anyone holding the line and making a point. They are much harder to hold."

Reenacting Stories

Children act out stories they have heard recently using position words such as *above*, *below*, *down*, *up*, *right*, *left*, *under*, *top*, *bottom*, *side*, *beside*, and *through.* Children either represent the characters in the stories themselves, or they model the stories by using storyboards.

The storyboards depict settings of recently studied themes or projects and can be used as backdrops for a variety of stories. Using objects to represent characters, children model stories by placing items appropriately on the storyboard. They use position words to describe the characters' actions.

"The Three Billy Goats Gruff" is a great story for children to perform; it is especially popular with prekindergartners. A small table can be used as a bridge. The narrator emphasizes many position words (such as *under*, *over*, *after*, *next*, *between*) and size words (such as *big*, *middle*, *little*) to describe the action as children act out the story. Afterward, the story can be rewritten with the characters in different locations (the troll on top of the bridge, the billy goats trotting under the bridge), which children often find amusing.

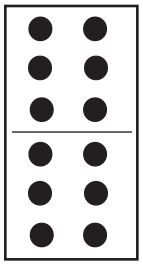
Kitchen Prints

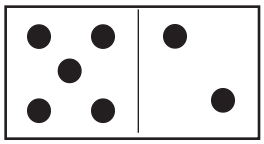
Children use a variety of kitchen utensils to make paint prints in the art center. The prints must fill up an entire piece of paper, but the individual prints on the paper may not touch one another. Forks, spatulas, handles, plastic lids, the tops of pans, cheese graters, slotted spoons, and food brushes create unique prints in a variety of shapes and partial shapes.

Children generally take great care in spacing the individual prints so they do not overlap. When finished, they describe the paint prints using shape and position words and have fun guessing which tools made the prints in each other's pictures.

Domino Flash

With half of it covered, the pip side of a large domino is quickly shown to the children. Based on their fleeting view of the pattern of the pips, children decide what number is displayed on that part of the domino. When children become comfortable with this activity, both halves of the domino face are shown and children must add the two amounts. (It is best to use a domino set with "double-six"—six pips on each half—as the highest value, as this activity may be too difficult for young children if higher values are used.)





Children quickly learn what the arrangement of pips for 1, 2, 3, 4, 5, and 6 look like. When asked how they recognize the numbers so quickly, their responses reveal their understanding of number as well as spatial sense. When Allen was asked how he knew five pips stood for 5, he said, "It's one in the middle and two up and two down. You don't even need to count—it's just there!" Jelani said that 2 was "so easy—there's one in one corner and one in the other corner. Easy!"

Making Frames

Children construct frames for three-dimensional shapes by bending pipe cleaners around objects found in the constructing center. (A section of the frame is lifted up to remove the object.) The frames are then displayed and children try to guess which shapes match which frames.

This activity fosters an understanding of shape and space. Children enhance their knowledge of edges, faces, and vertices when they resolve the problems of "making this part straight" or "getting all the pieces to meet at one point."

Treasure Map

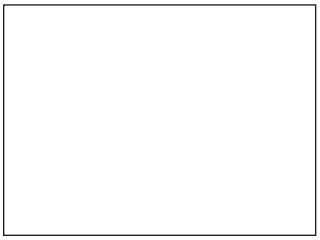
Eric Carle's book *The Secret Birthday Message* is a delightful story about a boy searching for his birthday surprise—a new puppy. The book uses many direction and shape words. A map at the end of the book visually represents story events, using shapes and arrows to indicate directions.

After listening to the story and studying the map, children make their own maps of the classroom, complete with shapes, arrows, and a hidden puppy. These wonderful maps can be created entirely using shapes. For example, the teacher's desk becomes a rectangle, the clock a circle, the doorway a rectangle, and the cabinet door a square. A puppy can be constructed by using two circles and cutting one in half to make ears.

When the classroom maps are finished, children "read" each other's to see if they can find the puppy. The maps are saved and often shared with other classes.

Quick Draw

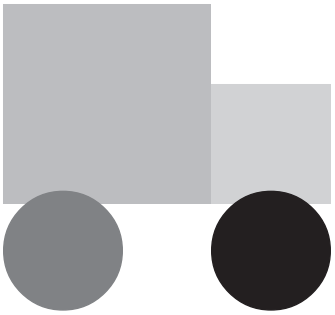
Children are briefly shown one or two shapes, which they quickly draw. Afterward, children discuss and describe the shapes or draw pictures using them. These pictures can be kept in a special "shape journal."



This activity is especially well suited to prekindergarten classes, although older children also enjoy it. After drawing a shape, children often create more pictures to put in their shape journals by drawing patterns featuring that shape. Other shapes can be included in the pictures as well. Some children prefer to draw the circle because "it's easy." Other shapes like hexagons are harder for children to draw, but after tracing cutouts of these shapes, children recognize them more readily and their drawings improve.

Shape Pictures

In this excellent learning center activity, children compose pictures using specified numbers of circles, triangles, and rectangles. First, children cut multiples of these shapes in various sizes from construction paper. Using a spinner divided into thirds (labeled *circle, triangle,* and *rectangle*) and a die with pips from 1 through 6, children spin the spinner and roll the die, select the appropriate number of the shapes indicated, and repeat the process once more. They then make pictures using the shapes they selected. Children display their pictures and describe their constructions using shape and position words.



The pictures can be quite interesting. Squares become animals, houses, or robots. Circles are often used as parts of flowers, the sun, or spots on a dog. Triangles become people's heads, tops of buildings, or race cars.

Secret Socks

Children create their own "secret socks," which contain several mystery shapes. A child threads two or three shape beads onto a pipe cleaner, folding the ends so the beads will not fall off. He places the pipe cleaner and beads in a secret sock. The secret socks are then traded. Children feel the shapes inside the socks *without* looking, and in their own words, they describe what they think is inside.

Children's descriptions are typically connected to everyday objects. They describe the sphere as a round ball, the cube as a box, and the cylinder as round with flat like a can. Using another pipe cleaner and shape beads, the children then make a copy of the shapes they believe are inside. Finally, answers are checked by opening the secret socks. The teacher may need to explain that beads of the same shape match even if they are different colors.

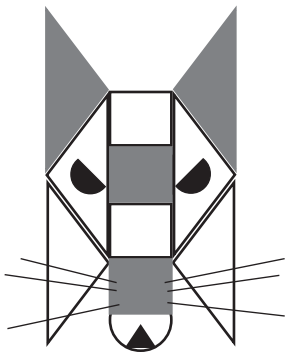
Mystery Shapes

The teacher or a child hides a three-dimensional shape in a box. The teacher or child gives clues, and children try to guess the hidden shape. For example, a child's clues for a rectangular block might include, "It's shaped like a cereal box. What shape do I have?"

Tangram Creations

Using one or more sets of construction paper tangram pieces, children can create original designs or follow pictures from tangram shape books. Pictures are named, described, and displayed for everyone in the class to see. This is a great activity for 4-year-olds, as long as they have enough time to experiment with the shapes.

Picture Pie Books



Ed Emberley's series of *Picture Pie* books are filled with pictures made from circles, squares, and partial shapes. He gives step-by-step directions for making insects, puppies, flowers, letters of the alphabet, and a variety of real and pretend characters.

Children view the directions and make the pictures of their choice, tracing the shapes needed or following the templates provided in the book. The pictures are easy to adapt or simplify. When associated with a particular theme or story the children encounter in any part of the curriculum, these pictures help them make connections between mathematics and other areas—social studies, science, art, and so on.

Bubble Wands

Children bend pipe cleaners into bubble wands of different shapes, predict what shapes the bubbles blown with these wands will be, and then blow bubbles. How surprising that all the bubbles are spheres! The children try over and over again to make cube or pyramid bubbles. Alas, bubbles are always spheres!

This activity provides useful experience in constructing shapes and recognizing spheres, but at this level the children are not given any scientific explanations of this phenomenon.

Aka Backa Soda Cracker

The Aka Backa Soda Cracker game helps children learn to recognize shapes and pass an object from right to left. It may also help the teacher to assess children's understanding of shape words.

Children sit in a circle and pat their legs in time to "Aka Backa Soda Cracker." The song is sung "Aka backa soda cracker, Aka Backa BOO [at *BOO,* everyone claps his hands in the air], Aka backa soda cracker, pass to you!" After learning the words and rhythm of the song, each child gets a manipulative shape. The teacher models how to pass from right to left. (The teacher may also wish to explain that moving from right to left inside the circle is like the movement of the hands of a clock, and that this circular direction is called *clockwise)*. When the words "Pass to you!" are sung, children pass their shape to the person on their left and receive a new shape from the person on their right. If done correctly, all pieces move one at a time in a clockwise direction. The game continues for as long as the class wishes.

Once the passing motion is error-free, the children are ready for a modification of the game. A leader for the activity is selected. Periodically, the leader says STOP at the end of a verse and then states the name of a shape. Anyone who has that shape holds it up in the air. Using descriptive terms like *side* and *corner*, the children holding that shape trace around it and describe it in unison, led by the leader. Those who do not have the shape hold their fingers up in the air and trace an imaginary shape.

Teachers should not teach this game too quickly, or the children may not master the passing motion. If passing from right to left is modeled step-by-step, then children learn it easily and never seem to tire of the game.

Creating a New Playground

Children in second grade design their ideal playground by constructing a model using blocks and other materials. The children draw building plans for their model, using a key of their own design with symbols that can be easily interpreted. Symbols might include a green rectangle to indicate a wooden plank or a red line to represent a metal bar. This activity is a good early exercise in symbolic reasoning.

Bubble Windows

In this popular activity, children from quadrilaterals from string and straws. After dipping these frames in a bubble solution of liquid soap and water, children investigate planes as well as the many different shapes that can be made from two or more bubble windows that intersect. Because of the surface tension of soapy water, bubble windows can be combined in interesting and unusual ways and can be manipulated easily. The children make fascinating discoveries about shape properties and planes.

What Am I Seeing?

Three-dimensional shapes—pyramids, cubes, spheres, cylinders—are placed on the overhead projector and covered with a thin piece of paper. The children try to guess what shape is on the projector by looking at the image on the screen. If a dark circle is on the screen, children may guess that a cylinder is on the overhead projector, since they know that cylinders have circles on both ends. Shapes such as a pyramid are more challenging; a pyramid can be set on its square base or on a triangular side on the projector, so a triangle or a square may appear on the screen.

This activity for learning about the relationship between two- and three-dimensional shapes is generally appropriate for children in second grade, or perhaps at the end of first grade. Children are able to see that three-dimensional shapes are made up of *faces* of different geometric shapes.

Straw Towers

Groups of children are challenged to make a tower one meter tall that can withstand a hurricane and an earthquake rated 7.5 on the Richter scale. The hurricane strikes when everyone blows as hard as possible on one side of the tower and then on the other side. The earthquake hits when the teacher shakes the tower's cardboard base as hard as possible seven-and-a-half times. The tower is made with a maximum of 25 straws, 10 paper clips, a foot of masking tape, and one cardboard base.

The teams design, build, and name their towers. The children are given many opportunities to try out their ideas before testing the towers against the elements. When one of the towers withstands the weather, the architects are very proud.

After analyzing the results, children typically hypothesize that triangular bases are the strongest. They begin to observe other buildings and structures to validate their findings.

Cube Constructions

Children are challenged to build different constructions from five cubes that attach to each other. There are 29 possible constructions. Each construction must be built so that it can be picked up, flipped, or moved in any way and not match any other construction. All cubes must be connected in each construction.

This is a good group problem-solving activity, as children can work in teams to make the different constructions. Children often find 29 different ways to connect the five cubes, but upon closer inspection the constructions they create may not all be different. Unique constructions are displayed so children can compare their efforts. This is an excellent task for persistent builders!

Principles in Action

The vignettes and other teaching examples in Chapter 6, "Geometry and Spatial Sense," reflect the curriculum, instruction, and assessment principles that form the basis for this book. To clarify how specific principles look in practice, this chart highlights four instances demonstrating selected principles.

Curriculum Principle 1 The mathematical content should be rich, varied, and relevant to children.	Shape and spatial sense are cultivated during the tangram activity, as are vocabulary words that might typically be introduced at more advanced grade levels (pp. 107–110).
Curriculum Principle 2 Essential mathematical processes are solving problems, reasoning, communicating, making connections, and representing.	Children's reasoning processes are emphasized when the teacher asks them to articulate their matching strategies for shape configurations during the tangram activity.
Curriculum Principle 4 Curriculum decisions should take into account children's knowledge, abilities, and interests.	A child-centered approach is demonstrated frequently during the tangram activity as the teacher uses children's vocabulary to describe shapes, values children's strategies for replicating shapes, and joins in the laughter when children play with the words squashed quadrilateral.
Instruction Principle 2 Interacting with children and promoting interactions between children are key roles of the early childhood teacher.	The teacher promotes children's interactions when he changes Kimberly's focus from copying to helping while emphasizing the importance of learning from others' shapes (p. 109).

Juanita V. Copley is an associate professor in curriculum and instruction at the University of Houston in Texas, where she is coordinator of the early childhood program and director of a professional development project known as the Early Childhood Collaborative. For the past 11 years, Dr. Copley has been actively involved in teacher education at the university and has taught in early childhood programs in the public school system. For 10 years she served as a mathematics/science instructional specialist in the Alief Independent School District, with responsibilities for prekindergarten through fifth grade.

Dr. Copley is editor of the book *Mathematics in the Early Years* (1999), copublished by the National Council of Teachers of Mathematics and the National Association for the Education of Young Children.

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